Sample Midterm Questions

Note: this is not a sample midterm; the midterm itself will have fewer questions. However, these are the type of questions you can expect on the midterm. The purpose is to give you lots of questions to practice.

1. (a) State (in words) what

$$\lim_{x \to a} f(x) = L$$

   means.

   (b) Define what it means for $f(x)$ to be continuous at a.

   (c) Define the derivative of $f(x)$ at a. (Either limit is acceptable).

2. Determine, for the following function f(x):

   (a) Where f(x) is not continuous.

   (b) Where f'(x) does not exist.

3. Find the slope of the secant line for $f(x) = 3x^2 - x$ from $x = 2$ to $x = 3$.

4. Find the value of the following limits:

   (a)

$$\lim_{x \to 2} x^3 - x^2 + 3x - 2$$

(b)

$$\lim_{x \to -2} \frac{x^2 - 4}{x^2 + 5x + 6}$$

(c)

$$\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3}$$

5. Find the derivative of the following functions *using the definition of the derivative*, and not any rules of differentiation.

   (a) $f(x) = x^3$ at $a = 1$.

   (b) $f(x) = \sqrt{x - 1}$ at $a = 3$.

   (c) $f(x) = \frac{1}{x+2}$ at $a = 4$.

6. Find the derivative of the following functions; you may use rules of differentiation for these questions. State which rule you are using (product rule, chain rule, etc) as you use it.

   (a) $f(x) = \sqrt{5x - 3}$

   (b) $f(x) = \cos^2(x)$

   (c) $f(x) = \sin(x^{2/3})$

   (d) $f(x) = \tan(x)e^{2x}$

   (e) $f(x) = \frac{x^2+2}{x^3+3}$

   (f) $f(x) = \arcsin(6x)$

   (g) $f(x) = \ln(x^{-4} + 10x^3)$

   (h) $f(x) = \sec(\ln(x))$

7. What is the equation of the tangent line to $f(x) = (x^2 + 3)(\sqrt{x} + 1)$ at $x = 4$?

8. The curve with equation $y^2 = 5x^4 - x^2$ is called a *kampyle of Eudoxus*. Find the slope of the tangent line to this curve at the point $(1, 2)$.

9. If a car has moved $\ln(t^3 - 6t^2 + 12t + 1)$ km after t hours, when is the car at rest?

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10. Newton's Law of Gravitation says that the magnitude F of the force exerted by a body of mass m on a body of mass M is

$$F = \frac{GmM}{r^2}$$

where G is the gravitional constant, and r is the distance between the bodies. Find the rate of change of F with respect to the distance r when $r = 2$, $m = 10$, and $M=3$.

11. Grain pouring from a chute at the rate of $1/4$ m^3/min forms a conical pile whose height h is always twice its radius r. How fast is the height increasing when the pile is $2m$ high? (Note: the volume of a conical pile is given by $V = \frac{1}{3}\pi r^2 h$).

12. Two cars start moving from the same point. One travels south at 60m/h, and the other travels west at 25m/h. How fast is the distance between the cars increasing two hours later?